

Optimization of geometrical parameters for Stirling engines based on theoretical analysis

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ABSTRACT

This study is aimed at theoretical analysis of the effects of the geometrical parameters on the shaft work of the Stirling engines. The optimal combination of the phase angle and the swept volume ratio, that leads to maximization of the shaft work of the engine, is obtained under different specified conditions. Effects of the effectiveness of mechanism, the dead volume ratio, and the temperature ratio on the maximum shaft work of the engine as well as the optimal combination of the phase angle and the swept volume ratio are evaluated. Theoretical analysis of the performance of three types of Stirling engines, α -, β -, and γ -type, has also been carried out, and a comparison in relative performance among these three types of engines is attempted. Results show that for the particular cases considered in this study, the β -type Stirling engine produces highest shaft work and the γ -type engine the lowest. In general, the γ -type engine must be very mechanism effective so as to deliver sufficient shaft work. However, among the three types of engines, the γ -type engine is most capable of operating with low temperature difference. On the contrary, the α -type engine is particularly not suitable for the applications with low temperature difference since its dimensionless shaft work is found to gradually vanish as the temperature ratio is increased.

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1. Introduction

Stirling engines are referred to as external combustion engines, and hence, they can be operated with a variety of heat sources and have been applied for energy conversion in a number of engineering applications [1–3]. The ideal regenerative Stirling cycle consists of two isothermal processes and two isochoric processes, as shown in Fig. 1. In theory, the thermal efficiency of an ideal regenerative Stirling cycle is equal to the Carnot cycle efficiency under the same thermal reservoirs temperatures. In practices, the Stirling engines are classified into three types in terms of configuration [4]. The schematics of the three types of Stirling engines and their varying displacements of the pistons/displacers are plotted in Fig. 2. In an α -type engine, the total volume of the working fluid in the engine is determined by the relative displacements of its two pistons, whereas in a β - or γ -type engine, the total volume of the working fluid in the engine is simply determined by the displacement of piston and is not influenced by the position of displacer.

According to the literature survey, the first successful analytical model for the Stirling engine was presented by Schmidt [5] in 1871. However, based on the Schmidt model, a group of researchers, for example, Finkelstein [6], Walker [7], and Kirkley [8], investigated the optimal phase angle and swept volume ratio for the

α -, β -, and γ -type Stirling engines and refrigerators. It appears in these previous studies that among these three types of engines, the β -type Stirling engine produces highest indicated work whereas the γ -type the lowest.

Lately, the non-isothermal condition was introduced to the analysis of Stirling cycle by Finkelstein [9]. In recent years, theoretical analysis was extended to the irreversible cycle analysis [10–14], and some numerical models for Stirling engine were also developed. Martaj et al. [14] presented a thermodynamic analysis of a low temperature Stirling engine at steady state operation, and energy, entropy and exergy balances were presented at each main element of the engine. Recently, Timoumi et al. [15,16] and Tlili et al. [17] optimized the regenerator and swept volume of the Stirling engine by using numerical model. Cheng and Yu [18] proposed a thermodynamic model for the β -type Stirling engine which taking into account the effectiveness of the regenerative channel as well as the thermal resistances of the heating and cooling heads. Lately, the thermodynamic model was then incorporated with a dynamic model to perform a dynamic simulation model for the β -type Stirling engines with cam-drive mechanisms by the same group of authors [19]. More recently, Yang et al. [20] applied thermoacoustic theory to analyze a free-piston Stirling engine.

Note that the regenerator effectiveness has been recognized as an important factor to the performance of the Stirling engines. The regenerator effectiveness is dependent on porosity, permeability and material of porous matrix. Temperature distribution in the

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Nomenclature

E	effectiveness of mechanism
m	mass of working fluid (kg)
p	pressure of work space (kPa)
p_{ac}	average cyclic pressure, $=\sqrt{p_{\max}p_{\min}}$ (kPa)
p_b	back pressure (kPa)
p_{mean}	average pressure, $=(p_{\max} + p_{\min})/2$ (kPa)
$\bar{Q}_{in}$	dimensionless heat input
R	gas constant (kJ)/(kg K)
r	compression ratio, $=V_{\max}/V_{\min}$
T_c	temperature of compression space (K)
T_d	temperature of dead space, $=(T_e + T_c)/2$ (K)
T_e	temperature of expansion space (K)
V	volume variation of total working space (m^3)
V_c	volume variation of compression space (m^3)
V_{cs}	piston swept volume (m^3)
V_d	volume of dead space (m^3)
V_e	volume variation of expansion space (m^3)
V_{es}	displacer swept volume (m^3)
V_r	volume of regenerator (m^3)
V_T	total swept volume, $=V_{cs} + V_{es}$ (m^3)
W	indicated work (kJ)
W_s	shaft work (kJ)

W_-	forced work (kJ)
$\bar{W}$	dimensionless indicated work
$\bar{W}_s$	dimensionless shaft work
$\bar{W}_-$	dimensionless forced work

Greek Symbols

α	phase angle ($^\circ$)
η	thermal efficiency
κ	swept volume ratio, $=V_{cs}/V_{es}$
τ	temperature ratio, $=T_c/T_e$
ϕ	crank angle ($^\circ$)
χ	dead volume ratio, $=V_d/V_{es}$

Subscripts

ac	cyclic average
c	compression space
d	dead space
e	expansion space
max	maximum
mean	average
min	minimum
r	regenerator

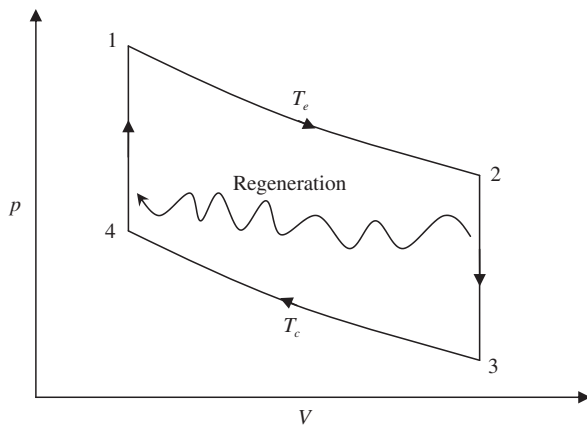


Fig. 1. Ideal regenerative Stirling cycle.

regenerator can be measured by experiment to determine the regenerator effectiveness. Furthermore, the magnitude of the regenerator effectiveness is also dependent on the operating speed. Actually, they decrease with an increasing operating speed. This is because the time for heat exchange between the gas and the porous matrix of the regenerator is decreased by increasing the operating speed.

In addition to the above thermodynamic models, a three-dimensional computational fluid dynamics (CFDs) approach may also be adopted in the analysis of the Stirling engines. The CFD simulation of a solar-powered Stirling engine was performed by Mahkamov [21]. Numerical predictions of the flow, pressure and thermal fields in a reciprocating piston-cylinder assembly of a Stirling engine were carried out by means of the CFD methods by Cheng and Hung [22]. It has been recognized that the CFD methods give a certain amount of detailed information that are not easily obtained by the experiments. However, the three-dimensional CFD computation is in general rather time consuming and requires much more computation efforts than the thermodynamic models.

On the other hand, experiments have also been conducted by a group of researchers. To name a few, Batmaz and Ustun [23] designed an α -type Stirling engine and measured the power at different heating temperature and charged pressure. Cinr et al. [24] and Karabulut et al. [25] performed experimental study on the development of a β -type Stirling engine working at different charged pressure and heating temperature. Kongtragool and Wongwises [26] conducted experiments for the low temperature differential γ -type Stirling engines. In these experimental studies, the engine torque, the shaft work, and the thermal efficiency at different engine speeds were measured.

Senft [27,28] considered the effect of the mechanical loss and proposed a concept of the mechanism effectiveness. The mechanism effectiveness is well known as a ratio measure of how well a mechanism utilizes work if it is regarded as a work transmitter. The value of mechanism effectiveness can be estimated by using the dynamic model similar to that proposed by Cheng and Yu [19]. As stated in Refs. [27,28], the typical value of the mechanism effectiveness is within the range between 0.7 and 0.9. In the concept, the shaft work of engine can be determined in terms of the indicated work and the forced work. In addition, the indicated work is calculated simply by integrating the area inside the cycle in a p - V diagram and the forced work is determined by calculating the areas of the shaded zones shown in Fig. 3, where p_b is the back pressure exerted on the opposite side of the piston. The back pressure could usually be the atmospheric pressure or an elevated pressure in case a pressurized space is placed on the opposite side of the piston as a back zone.

The fundamental problems that the designers of the Stirling engines have to face are the time consumption in the design process and the cost of precision manufacturing. In most of the cases, the determination of the geometrical parameters of the engines is based on the experience of the designers or on insufficient costly experimental information. Therefore, the improvement of the engine can only be carried out in a relatively high-cost, time-consuming trial-and-error process. In this regard, modeling the engines is expected to be one of the alternative solutions to these issues. Furthermore, it is important to mention that the shaft work is exactly the output work delivered by the engine. However, in most of these

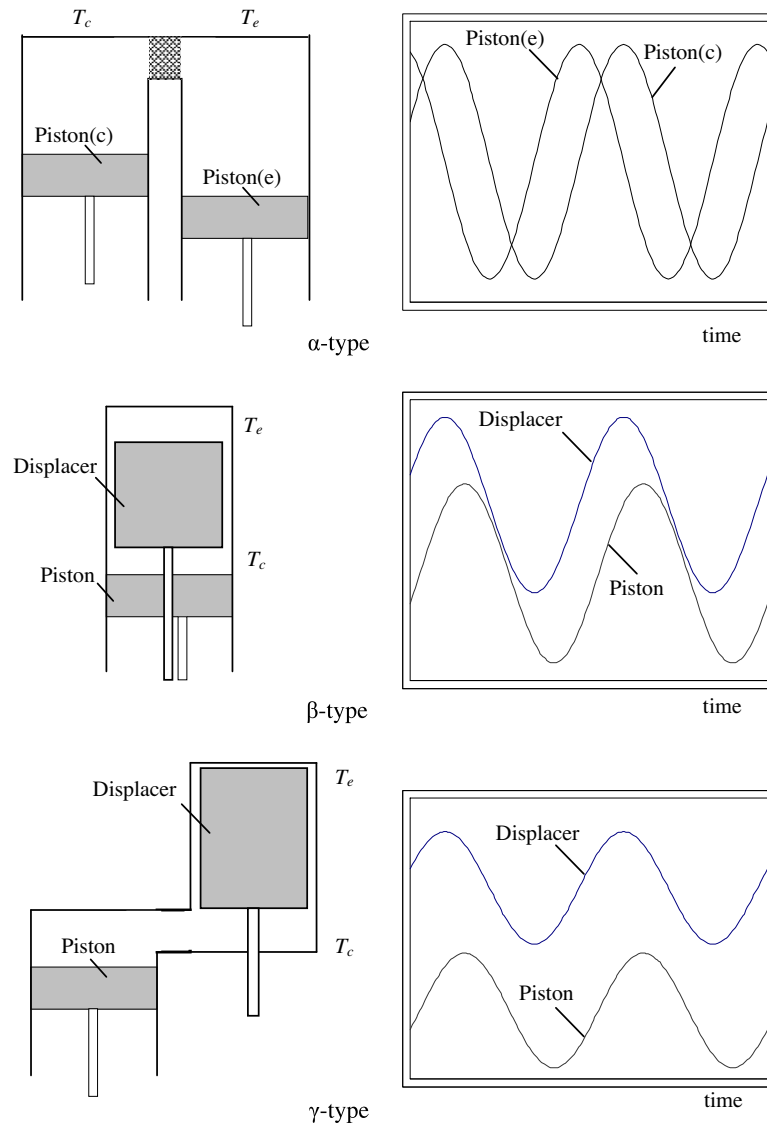


Fig. 2. α -, β -, and γ -type Stirling engines and their varying displacements of pistons and displacers.

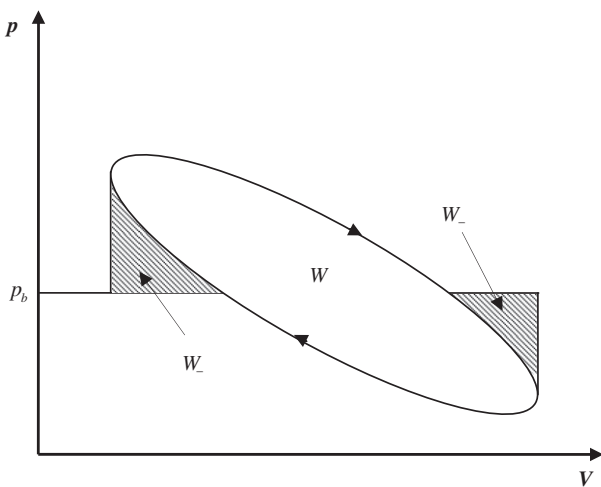


Fig. 3. Indicated work and forced work [27,28].

existing models [5–22] it is the indicated work that was maximized, not the shaft work. Therefore, the performance of the engine might

have not been fairly assessed. Under these circumstances, the present study is aimed at optimization of the geometrical parameters for the Stirling engines by introducing the concept of the mechanism effectiveness into the theoretical model. The optimal combination of the phase angle and the swept volume ratio, that leads to maximization of the shaft work delivered by the engine, is then investigated. Meanwhile, a comparison in the relative performance of the α -, β -, and γ -type engines is attempted.

2. Theoretical analysis

Here, the reversible model is adopted. A simple idealized model enables researchers to study the effects of major parameters which dominate the cycle without getting bogged down in the details. The present study is somewhat idealized, but they still retain the general characteristics of the real cycles they represent.

In this study, a group of dominant parameters are taken into account. They are: mechanism effectiveness (E), phase angle (α), swept volume ratio (κ), dead volume ratio (χ), and temperature ratio (τ). The first four parameters are referred to as the geometrical variables, and the temperature ratio is an operating parameter.

During the operation of the Stirling engine, the volume variation of the expansion space should move ahead of the compression space by a leading time. Hence, there exists a phase angle (α) between the movements of expansion and compression pistons (α type) or between displacer and piston (β and γ types).

In addition, the swept volume ratio, the dead volume ratio and the temperature ratio are defined as

$$\kappa = V_{cs}/V_{es}, \quad \chi = V_d/V_{es}, \quad \text{and} \quad \tau = T_c/T_e \quad (1)$$

respectively, where V_{cs} , V_{es} , and V_d denote the piston swept volume, the displacer swept volume, and the dead volume. The dead volume (V_d) of the engine is determined by $V_d = V_{emin} + V_{cmin} + V_r$, where V_{emin} and V_{cmin} are the clearance volumes of the expansion and compression spaces, respectively, and V_r is the volume of regenerator channels.

Dimensionless shaft work, dimensionless indicated work and dimensionless forced work are defined, respectively, as

$$\overline{W}_s = \frac{W_s}{mRT_e}, \quad \overline{W} = \frac{W}{mRT_e}, \quad \overline{W}_- = \frac{W_-}{mRT_e} \quad (2)$$

The dimensionless parameters are based on hot temperature and are different from those using average pressure and volume, and hence they are more accurate. When the dimensionless shaft work is fixed, more mass of the working fluid or higher hot temperature means higher shaft work. Note that the dimensionless shaft work ($\overline{W}_s$) is the objective function to be maximized in the present study.

Theoretical analyses of different types of Stirling engines are described individually in the following:

In an α -type Stirling engine, the volumes of the expansion and the compression spaces can be expressed as

$$V_e(\phi) = V_{emin} + \frac{1}{2}V_{es}(1 + \sin(\phi + \alpha)) \quad (3a)$$

$$V_c(\phi) = V_{cmin} + \frac{1}{2}V_{es}\kappa(1 + \sin \phi) \quad (3b)$$

For the β -type or the γ -type Stirling engine, the relation of the volume variation of the expansion space is similar to that for the α -type engine, which is dependent on the displacement of the displacer only. However, the volume of the compression space is dependent on the displacements of both displacer and piston. That is, for the β -type engine:

$$V_e(\phi) = V_{emin} + \frac{1}{2}V_{es}(1 + \sin(\phi + \alpha)) \quad (4a)$$

$$V_c(\phi) = V_{cmin} + \frac{1}{2}V_{es}(\sqrt{1 - 2\kappa \cos \alpha + \kappa^2} + \kappa \sin \phi - \sin(\phi + \alpha)) \quad (4b)$$

and for the γ -type engine:

$$V_e(\phi) = V_{emin} + \frac{1}{2}V_{es}(1 + \sin(\phi + \alpha)) \quad (5a)$$

$$V_c(\phi) = V_{cmin} + \frac{1}{2}V_{es}(1 + \kappa + \kappa \sin \phi - \sin(\phi + \alpha)) \quad (5b)$$

Then, the total volume (V) can be calculated with

$$V(\phi) = V_e(\phi) + V_c(\phi) + V_r = V_{es}\Phi(\phi) \quad (6)$$

where

$$\Phi(\phi) = \chi + \frac{1}{2}(1 + \kappa) + \frac{1}{2}(\sin(\phi + \alpha) + \kappa \sin \phi) \quad \text{for } \alpha \text{ type} \quad (7a)$$

$$\Phi(\phi) = \chi + \frac{1}{2}\left(1 + \sqrt{1 - 2\kappa \cos \alpha + \kappa^2}\right) + \frac{1}{2}\kappa \sin \phi \quad \text{for } \beta \text{ type} \quad (7b)$$

$$\Phi(\phi) = \chi + \frac{1}{2}(2 + \kappa) + \frac{1}{2}\kappa \sin \phi \quad \text{for } \gamma \text{ type} \quad (7c)$$

The compression ratio of the engine is determined in terms of the maximum and the minimum values of the total volume as

$$r = \frac{V_{\max}}{V_{\min}} = 1 + \frac{2\sqrt{1 + 2\kappa \cos \alpha + \kappa^2}}{2\chi + 1 + \kappa - \sqrt{1 + 2\kappa \cos \alpha + \kappa^2}} \quad \text{for } \alpha \text{ type} \quad (8a)$$

$$r = \frac{V_{\max}}{V_{\min}} = 1 + \frac{2\kappa}{2\chi + 1 - \kappa + \sqrt{1 - 2\kappa \cos \alpha + \kappa^2}} \quad \text{for } \beta \text{ type} \quad (8b)$$

$$r = \frac{V_{\max}}{V_{\min}} = 1 + \frac{\kappa}{1 + \chi} \quad \text{for } \gamma \text{ type} \quad (8c)$$

According to the information provided in Refs. [13,18,19,28], the pressure drop between the working spaces in the Stirling engine is negligible. Therefore, the pressure is assumed to be uniform throughout the engine and can be calculated in terms of the volumes and the temperatures of the working spaces as

$$p = \frac{mR}{\frac{V_e}{T_e} + \frac{V_c}{T_c} + \frac{V_d}{T_d}} \quad (9)$$

where the temperature of dead space is approximated by $T_d = (T_c + T_e)/2$. The pressure can be further expressed in the following form:

$$p = \frac{mRT_e}{V_{es}} \Psi(\phi, \alpha, \kappa, \tau, \chi) \quad (10)$$

where

$$\Psi(\phi, \alpha, \kappa, \tau, \chi) = \left[\frac{2\chi}{(1 + \tau)} + \frac{\kappa + \tau}{2\tau} + \frac{\kappa \sin \phi}{2\tau} + \frac{\sin(\phi + \alpha)}{2} \right]^{-1} \quad \text{for } \alpha \text{ type} \quad (11a)$$

$$\Psi(\phi, \alpha, \kappa, \tau, \chi) = \left[\frac{2\chi}{(1 + \tau)} + \frac{\tau + \sqrt{1 - 2\kappa \cos \alpha + \kappa^2}}{2\tau} + \frac{\kappa \sin \phi}{2\tau} + \frac{(\tau - 1)\sin(\phi + \alpha)}{2\tau} \right]^{-1} \quad \text{for } \beta \text{ type} \quad (11b)$$

$$\Psi(\phi, \alpha, \kappa, \tau, \chi) = \left[\frac{2\chi}{(1 + \tau)} + \frac{1 + \tau + \kappa}{2\tau} + \frac{\kappa \sin \phi}{2\tau} + \frac{(\tau - 1)\sin(\phi + \alpha)}{2\tau} \right]^{-1} \quad \text{for } \gamma \text{ type} \quad (11c)$$

Combining Eqs. (2), (6), (7a), (10) and (11) leads to the dimensionless indicated work as

$$\overline{W} = \frac{\oint p dV}{mRT_e} = \int_0^{2\pi} \Psi \frac{d\Phi}{d\phi} d\phi \quad (12)$$

And the dimensionless heat input can be calculated by

$$\overline{Q}_{in} = \frac{\oint p dV_e}{mRT_e} = \frac{1}{2} \int_0^{2\pi} \Psi \cos(\phi + \alpha) d\phi \quad (13)$$

Integrating the terms on the right-hand side of Eqs. (12) and (13) by using residual theorem, one obtains the dimensionless indicated work and dimensionless heat input of the three types of Stirling engine as

$$\overline{W} = \frac{2\pi\tau\kappa(1 - \tau) \sin \alpha}{a^2 + b^2} \left(\frac{\beta - \sqrt{\beta^2 - (a^2 + b^2)}}{\sqrt{\beta^2 - (a^2 + b^2)}} \right) \quad (14a)$$

$$\overline{Q}_{in} = \frac{2\pi\tau\kappa \sin \alpha}{a^2 + b^2} \left(\frac{\beta - \sqrt{\beta^2 - (a^2 + b^2)}}{\sqrt{\beta^2 - (a^2 + b^2)}} \right) \quad (14b)$$

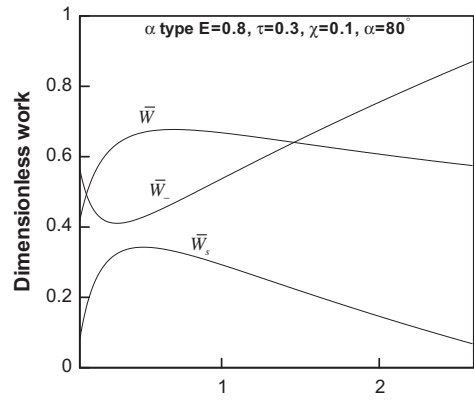
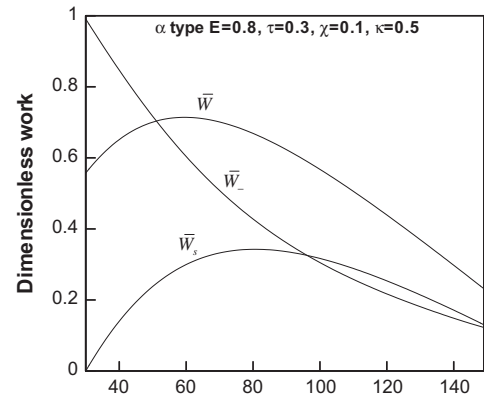
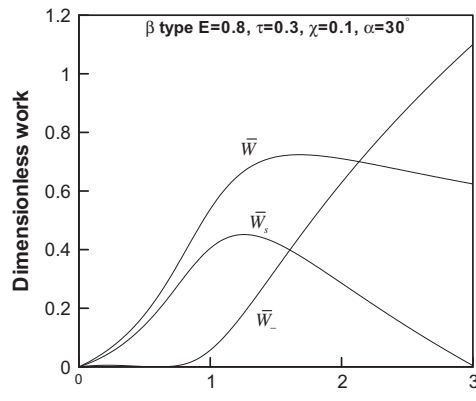
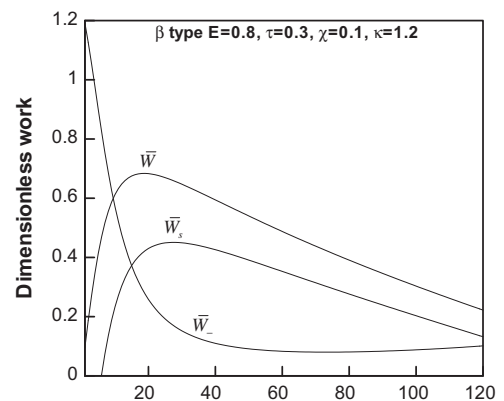
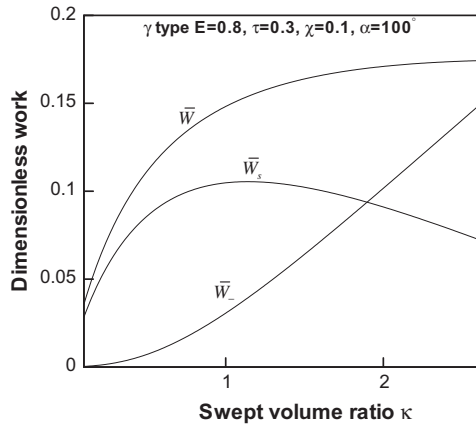
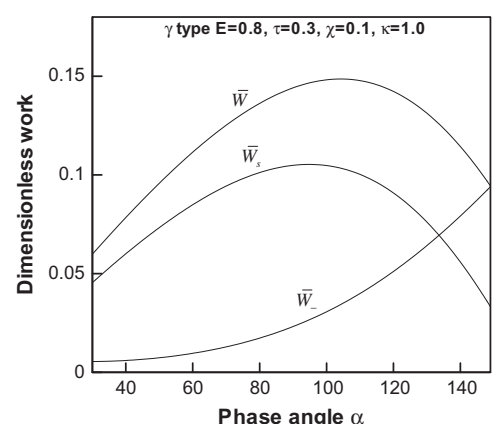
(a) α type with $\alpha=80^\circ$ (a) α type with $\kappa=0.5$ (b) β type with $\alpha=30^\circ$ (b) β type with $\kappa=1.2$ (c) γ type with $\alpha=100^\circ$ (c) γ type with $\kappa=1.0$

Fig. 4. Effects of swept volume ratio on dimensionless works, at $E=0.8$, $\tau=0.3$, and $\chi=0.1$.

where for α type:

$$\begin{aligned} a &= \tau \sin \alpha \\ b &= \kappa + \tau \cos \alpha \\ \beta &= 4\tau\chi/(1+\tau) + \kappa + \tau \end{aligned} \quad (15a)$$

for β type:

$$\begin{aligned} a &= -(1-\tau) \sin \alpha \\ b &= \kappa - (1-\tau) \cos \alpha \\ \beta &= 4\tau\chi/(1+\tau) + \tau + \sqrt{1-2\kappa \cos \alpha + \kappa^2} \end{aligned} \quad (15b)$$

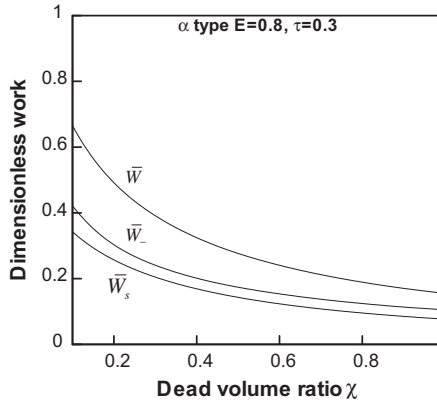
Fig. 5. Effects of phase angle on dimensionless works, at $E=0.8$, $\tau=0.3$, and $\chi=0.1$.

for γ type:

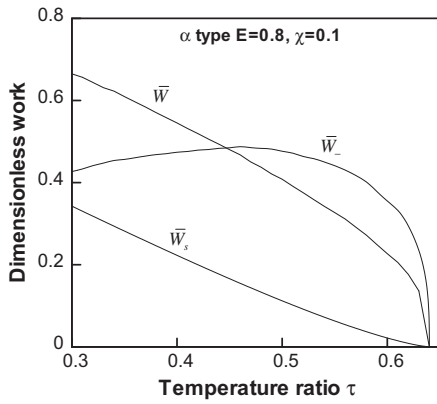
$$\begin{aligned} a &= -(1-\tau) \sin(\alpha) \\ b &= \kappa - (1-\tau) \cos(\alpha) \\ \beta &= 4\tau\chi/(1+\tau) + 1 + \tau + \kappa \end{aligned} \quad (15c)$$

Note that the magnitudes of the forced work and the indicated work are dependent on the back pressure (p_b). As discussed by Senft [29], the optimal value of the back pressure is the average cyclic pressure, which is

$$p_b = \sqrt{p_{\min} p_{\max}} \quad (16)$$



(a) Effects of dead volume ratio, at $E=0.8$, $\alpha=80^\circ$, $\kappa=0.5$, and $\tau=0.3$.



(b) Effects of temperature ratio, at $E=0.8$, $\alpha=80^\circ$, $\kappa=0.5$, and $\chi=0.1$.

Fig. 6. Effects of dead volume ratio and temperature ratio on dimensionless works for α -type engine.

where p_{min} and p_{max} are the minimum and maximum values of the pressure in a cycle. Introducing Eqs. (10) and (11) into the above equation leads to

$$p_b = \frac{mRT_e}{V_{es}} \sqrt{\Psi_{min} \Psi_{max}} \quad (17)$$

where

$$\sqrt{\Psi_{min} \Psi_{max}} = \frac{2\tau}{\sqrt{\beta^2 - (a^2 + b^2)}} \quad (18)$$

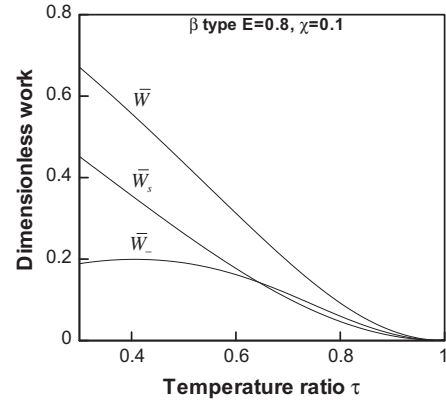
One can calculate the dimensionless shaft work for the engines as described by [28,29], which can be written in dimensionless form as

$$\bar{W}_s = E\bar{W} - \left(\frac{1}{E} - E\right)\bar{W}_- \quad (19)$$

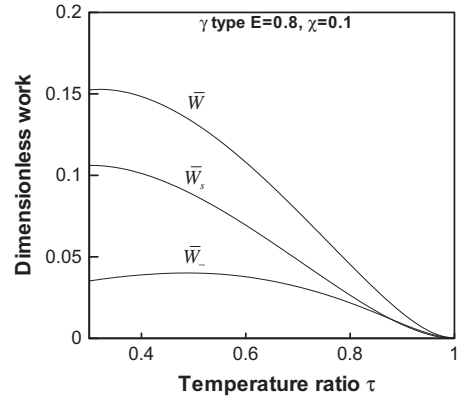
where

$$\bar{W}_- = \bar{W}_{e-} + \bar{W}_{c-} \quad (20)$$

and the dimensionless forced works done by the pistons in the expansion and the compression spaces for different types of engines can be determined as follows:



(a) β type with $E=0.8$, $\alpha=30^\circ$, $\kappa=1.2$, and $\chi=0.1$.



(b) γ type with $E=0.8$, $\alpha=100^\circ$, $\kappa=1.0$, and $\chi=0.1$.

Fig. 7. Effects of temperature ratio on dimensionless works for β - and γ -type engines.

2.1. α Type

Both the two pistons in the expansion and the compression spaces of the α type Stirling engine keep contact with the back pressure; therefore, one needs to consider the forced works done by the two pistons, which are

$$W_{e-} = \oint [(p - p_b)dV_e]^- = \frac{mRT_e}{2} \oint [(\Psi - \sqrt{\Psi_{min} \Psi_{max}}) \cos(\phi + \alpha) d\phi]^- \quad (21a)$$

$$W_{c-} = \oint [(p - p_b)dV_c]^- = \frac{mRT_e}{2} \oint [(\Psi - \sqrt{\Psi_{min} \Psi_{max}}) \kappa \cos \phi d\phi]^- \quad (21b)$$

where the superscripted negative sign means

$$[n]^{-1} = \begin{cases} 0 & \text{if } n \geq 0 \\ n & \text{if } n < 0 \end{cases} \quad (21c)$$

In dimensionless forms, the above equations become

$$\bar{W}_{e-} = \frac{1}{2} \oint [(\Psi - \sqrt{\Psi_{min} \Psi_{max}}) \cos(\phi + \alpha) d\phi]^- \quad (22a)$$

$$\bar{W}_{c-} = \frac{1}{2} \oint [(\Psi - \sqrt{\Psi_{min} \Psi_{max}}) \kappa \cos \phi d\phi]^- \quad (22b)$$

2.2. β Type

One of the major differences between β and α types is that the β -type engine has only one piston that keeps contact with the back

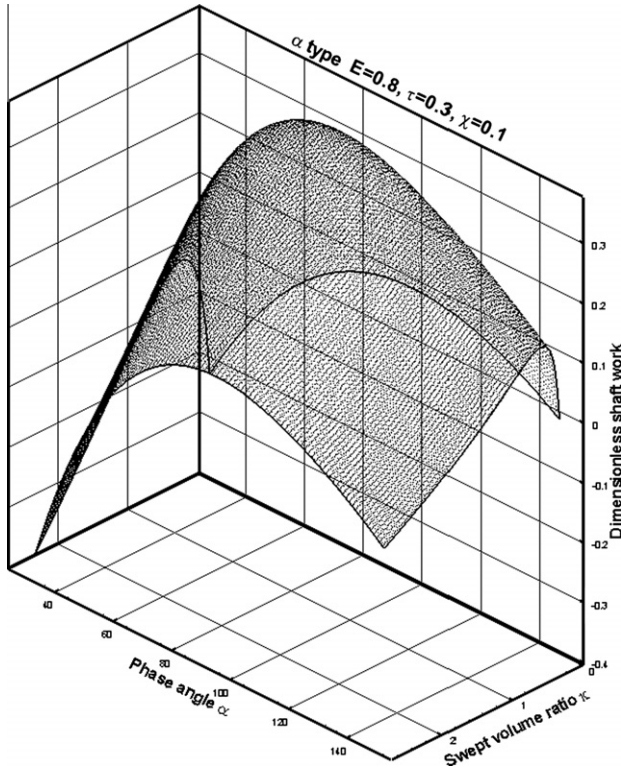


Fig. 8. Dimensionless shaft work as a function of phase angle and swept volume ratio for α -type engine at $E = 0.8$, $\tau = 0.3$, and $\chi = 0.1$.

pressure. Therefore, only a forced work done by the piston needs to be considered.

$$\bar{W}_- = \bar{W}_{c-} = \frac{1}{2} \oint \left[(\Psi - \sqrt{\Psi_{\min} \Psi_{\max}}) \kappa \cos \phi d\phi \right]^- \quad (23)$$

2.3. γ Type

The calculation of forced work for γ type is similar to that for β type since the γ -type engine also has only one piston in the compression space that keeps contact with the back pressure. Thus, the dimensionless forced work of γ type Stirling work can be calculated with Eq. (23).

3. Results and discussion

A parametric study is performed based on the theoretical analysis so as to evaluate the effects of the dominant parameters. Fig. 4 shows the effects of the swept volume ratio on the dimensionless works, at $E = 0.8$, $\tau = 0.3$, and $\chi = 0.1$, for all types of engines. Plotted in Fig. 4a are the results of $\bar{W}$, $\bar{W}_s$, and $\bar{W}_-$ for α -type Stirling engine with the phase angle $\alpha = 80^\circ$, Fig. 4b for β type with $\alpha = 30^\circ$, and Fig. 4c for γ type with $\alpha = 100^\circ$. It is clearly seen that for all types of engines the dimensionless shaft work first increases with the swept volume ratio in the low- κ regime. When the swept volume ratio is elevated, the shaft work reaches a maximum at a critical value of κ . Over the critical swept volume ratio, the shaft work gradually descends in the high- κ regime. The curves of dimensionless indicated work may also reach a maximum value; however, the peaks of the curves of the shaft work and the indicated work take place at different swept volume ratios. This implies that the optimal swept volume ratio obtained in terms of the indicated work is different from that in terms of the shaft work.

Note that the peak shaft work and its accompanying critical swept volume ratio are dependent on the specified values of α , E , τ , and χ .

Fig. 5 conveys the effects of the phase angle on the dimensionless works, at $E = 0.8$, $\tau = 0.3$, and $\chi = 0.1$. Fig. 5a shows the results for α -type Stirling engine with the swept volume ratio $\kappa = 0.5$, Fig. 5b for β type with $\kappa = 1.2$, and Fig. 5c for γ type with $\kappa = 1.0$. Again, a peak value of the dimensionless shaft work is observed for each of the α -, β -, and γ -type engines. In the three plots of this figure, it appears that the critical phase angle is approximately 80° for α type, 30° for β type, and 100° for γ type. These obtained optimal phase angles agree closely with the results provided by Kirkley [8] and Senft [28]. It is also noticed that for the particular cases considered in this study, the β -type Stirling engine produces highest shaft work and the γ -type engine the lowest.

Shown in Fig. 6 are the respective effects of the dead volume ratio and the temperature ratio on the dimensionless works of the α -type engine. Fig. 6a displays the influence of dead volume ratio at $E = 0.8$, $\alpha = 80^\circ$, $\kappa = 0.5$, and $\tau = 0.3$, and Fig. 6b shows the influence of temperature ratio, at $E = 0.8$, $\alpha = 80^\circ$, $\kappa = 0.5$, and $\chi = 0.1$. It is expected that as the dead volume in the engine is increased, the output work of the engine is reduced. Fig. 6a clearly reflects this expectation. It is observed that as the dead volume ratio is increased, the dimensionless indicated work and shaft works are both monotonically decreased.

In addition, as the temperature ratio is elevated, the temperature difference between the expansion and the compression spaces is actually reduced and as a result, the indicated work and the shaft work will be decreased. In Fig. 6b, it is noted that as the temperature ratio approaches 0.63, both the curves of $\bar{W}$ and $\bar{W}_-$ experience a sudden drop to zero, and meanwhile the dimensionless shaft work is found to gradually vanish before the temperature ratio approaches a limit of 0.63. This indicates that the α -type engine is inherently not suitable for the applications with low temperature difference. Note that the limiting value of the temperature ratio for the α -type engine is dependent on the E value.

Effects of temperature ratio on dimensionless works for β - and γ -type engines are displayed in Fig. 7. The cases considered in Fig. 7a are the β -type engines with $E = 0.8$, $\alpha = 30^\circ$, $\kappa = 1.2$, and $\chi = 0.1$, and in Fig. 7b the γ -type engines with $E = 0.8$, $\alpha = 100^\circ$, $\kappa = 1.0$, and $\chi = 0.1$. It is observed that the dimensionless works of the two types of engines are monotonically decreased with an increasing temperature ratio. Furthermore, as stated earlier, the α -type engine is not able to deliver shaft work at a temperature ratio elevated to 0.63. Nevertheless, in Fig. 7a and b, it is observed that for the β -type engine when the temperature ratio is elevated to 0.63, the shaft work still stays at approximately 33% of the work at $\tau = 0.3$, and the γ -type engine stays at 54%. This indicates that among the three types of engines, the γ -type engine is most capable of operating with low temperature difference even though its shaft work output is relatively low.

It is noticed that the temperature ratio τ represents the ratio of the temperature in compression space to that in expression space. Actually, the operating temperature ratio is dependent on the thermal resistance of heating and cooling spaces, the effectiveness of regenerator, the working fluid mass, and the engine speed [18].

In accordance with the parametric study discussed above, one may find that the shaft work of the engine is elevated when the dead volume ratio or the temperature ratio is decreased. However, the relation between the shaft work and the dead volume ratio or the temperature ratio is monotonical and hence, there is no need to optimize the dead volume ratio or the temperature ratio. On the other hand, the peak values of the shaft work can be observed as the phase angle or the swept volume ratio is varying. Therefore, optimization task of this study is focused on the two parameters. Fig. 8 conveys the dimensionless shaft work as a function of phase angle and swept volume ratio for

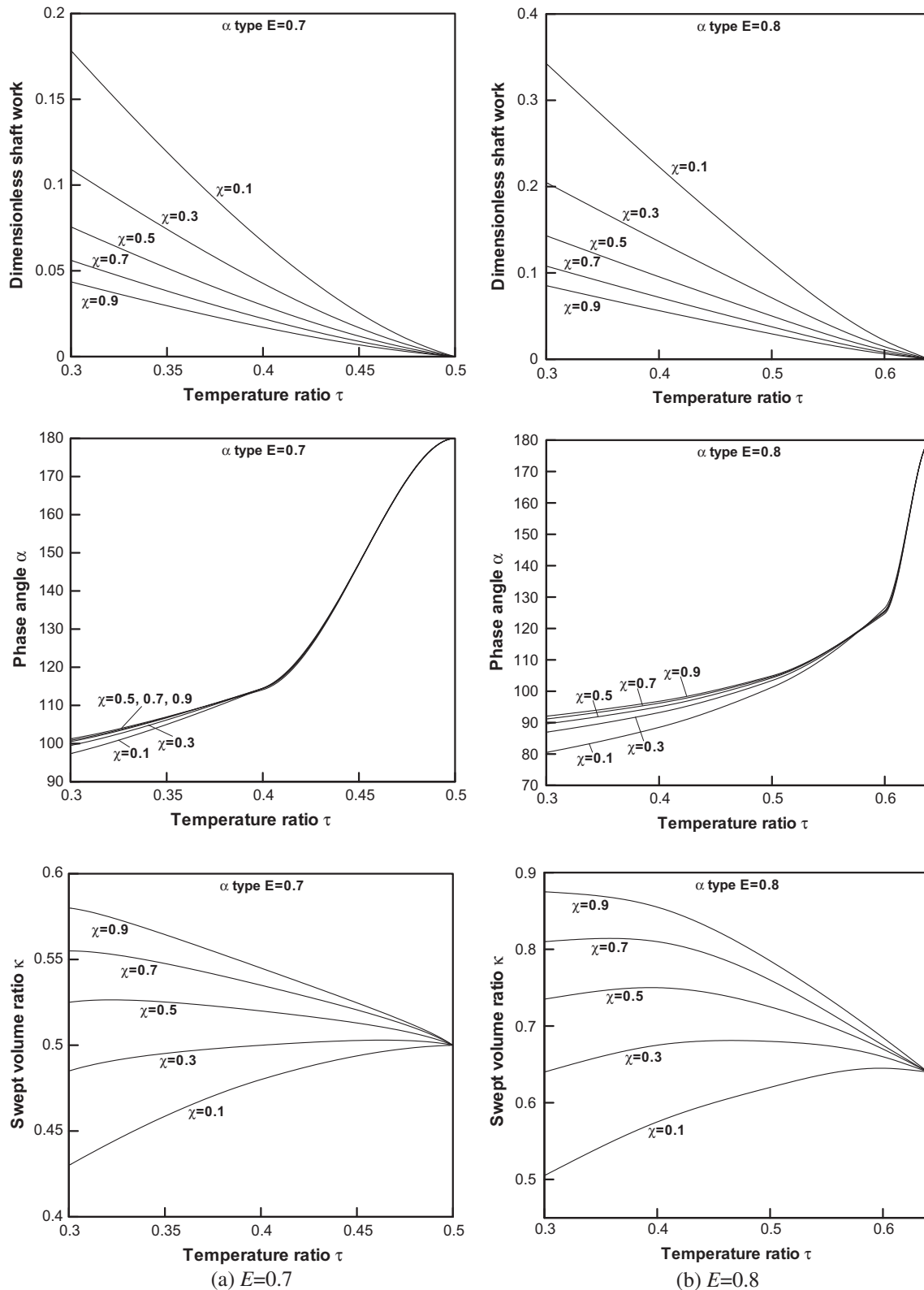


Fig. 9. Dependence of optimal $\bar{W}_s$, α , and κ on specified E , τ , and χ for α -type engine.

the α -type engine at $E=0.8$, $\tau=0.3$, and $\chi=0.1$. It is found that the dimensionless shaft work reaches the maximum of 0.343 at $\alpha=80.5^\circ$ and $\kappa=0.504$. However, it is important to mention that the optimal combination of the phase angle and the swept volume ratio and the maximum value of the dimensionless shaft

work are dependent on the specified conditions specified in terms of mechanism effectiveness, temperature ratio, and dead volume ratio. Once the specified condition is changed, the optimal geometrical parameters combination that results in the maximum shaft work is altered.

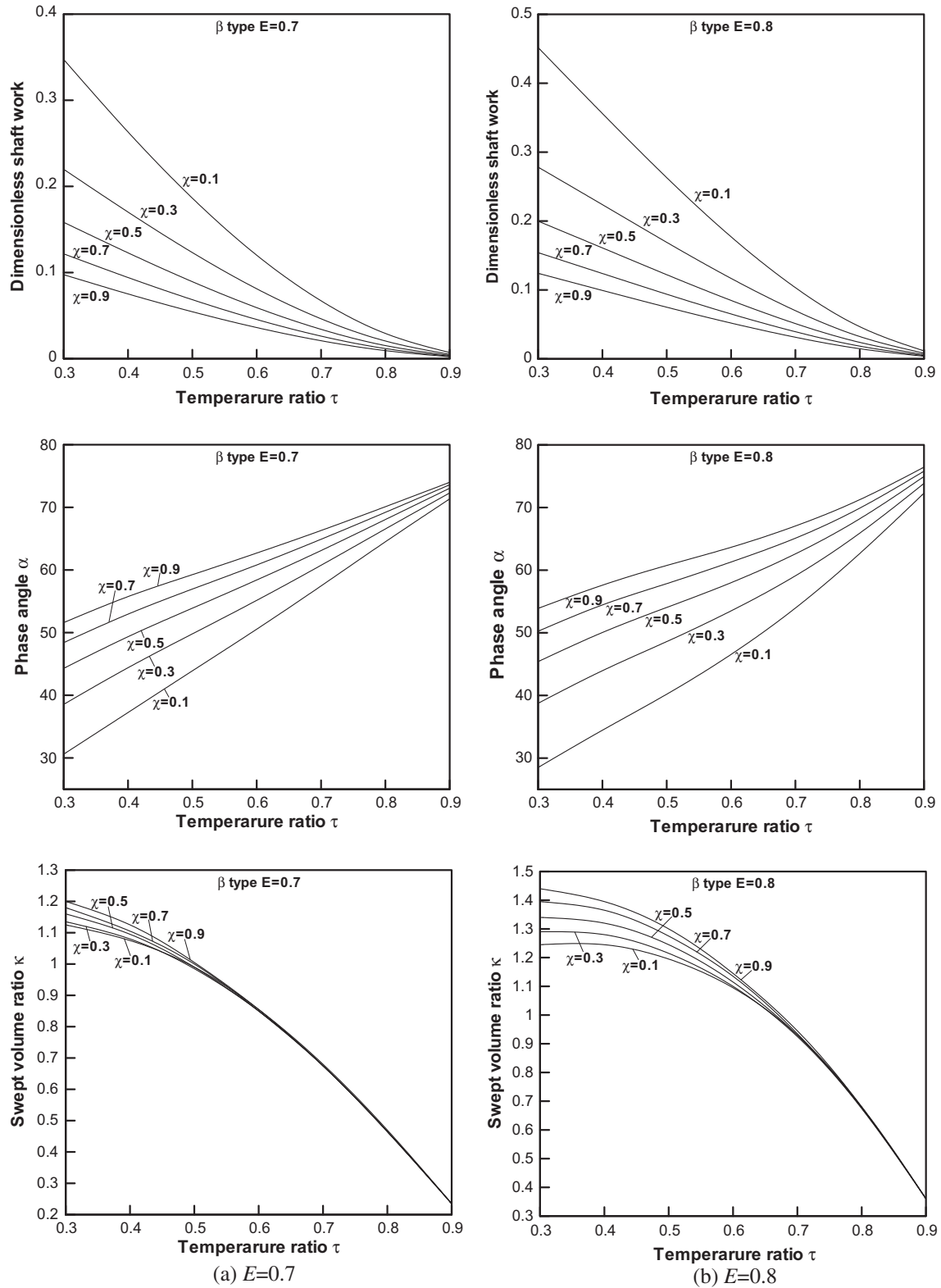


Fig. 10. Dependence of optimal $\bar{W}_s$, α , and κ on specified E , τ , and χ for β -type engine.

Fig. 9 conveys the dependence of the optimal values of the phase angle and the swept volume ratio and the maximum of the dimensionless shaft work on E , τ , and χ for the α -type engine. It is found that as the mechanism effectiveness is increased from 0.7 to 0.8, the maximum value of the dimensionless shaft work is nearly doubled. The dimensionless shaft work maximum is

decreased by increasing the value of τ or χ . Meanwhile, the optimal phase angle is elevated from 80° to 180° as the temperature ratio is varied from 0.3 to 0.6. In general, the optimal phase angle is decreased by reducing the value of τ or χ or by improving the mechanism effectiveness (E). The influence of χ on the optimal phase angle is more remarkable when E is higher. In addition,

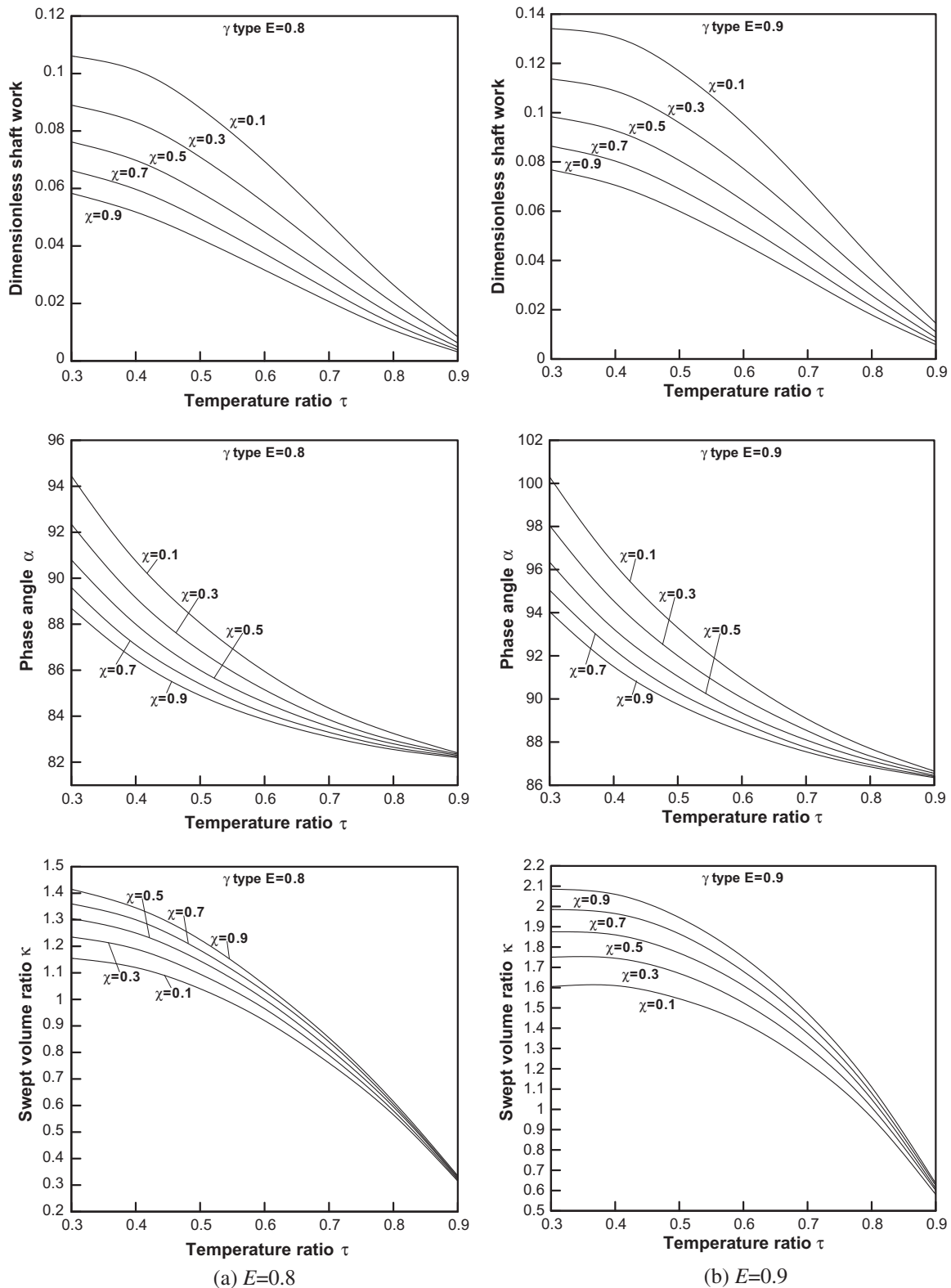


Fig. 11. Dependence of optimal $\bar{W}_s$, α , and κ on specified E , τ , and χ for γ -type engine.

the effects of the temperature ratio on the optimal swept volume ratio are more involved. At a lower dead volume ratio, say less than 0.3, the optimal swept volume ratio increases with the temperature ratio. However, as the dead volume ratio exceeds 0.5, the optimal swept volume ratio may be decreased by increasing the

temperature ratio. It is noted that when the mechanism effectiveness is improved, the optimal swept volume ratio is also increased.

Dependence of the optimal values of $\bar{W}_s$, α , and κ on specified E , τ , and χ for the β -type engine is illustrated in Fig. 10. Basically, the trend of the curves plotted in this figure is similar to those in Fig. 9.

The optimal shaft work delivered by the β -type engine is generally higher than that by the α -type engine. However, the optimal swept volume ratio is decreased by increasing the temperature ratio for all values of χ . Again, it is found that when the mechanism effectiveness is improved from 0.7 to 0.8, the performance of the engine is greatly increased.

Fig. 11 shows the optimal $\bar{W}_s$, α , and κ at different E , τ , and χ for the γ -type engine. In this figure, the mechanism effectiveness is assigned to be 0.8 and 0.9; however, it is noticed the optimal shaft work of the γ -type engine is still much less than those of the α - and the β -type engines shown in Figs. 9 and 10. This seemingly means that the γ -type engine must be very mechanism effective so as to deliver sufficient shaft work.

4. Concluding remarks

Theoretical analysis has been performed for the effects of the geometrical parameters on the shaft work of the Stirling engines. The optimal combination of the phase angle and the swept volume ratio toward maximization of the shaft work of the engine is investigated under different specified conditions. Effects of the effectiveness of mechanism, the dead volume ratio, and the temperature ratio on the maximum shaft work of the engine as well as the optimal combination of the phase angle and the swept volume ratio are evaluated. Theoretical analysis of the performance of three types of Stirling engines, α -, β -, and γ -type, has also been carried out, and a comparison in relative performance among these three types of engines is completed.

In accordance with the obtained results of this study, it is found that the β -type Stirling engine produces highest shaft work and the γ -type engine the lowest. In general, the γ -type engine must be very mechanism effective so as to deliver sufficient shaft work. However, among the three types of engines, the γ -type engine is most capable of operating with low temperature difference. On the other hand, the α -type engine is particularly not suitable for the applications with low temperature difference since its dimensionless shaft work is found to vanish at a higher temperature ratio.

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